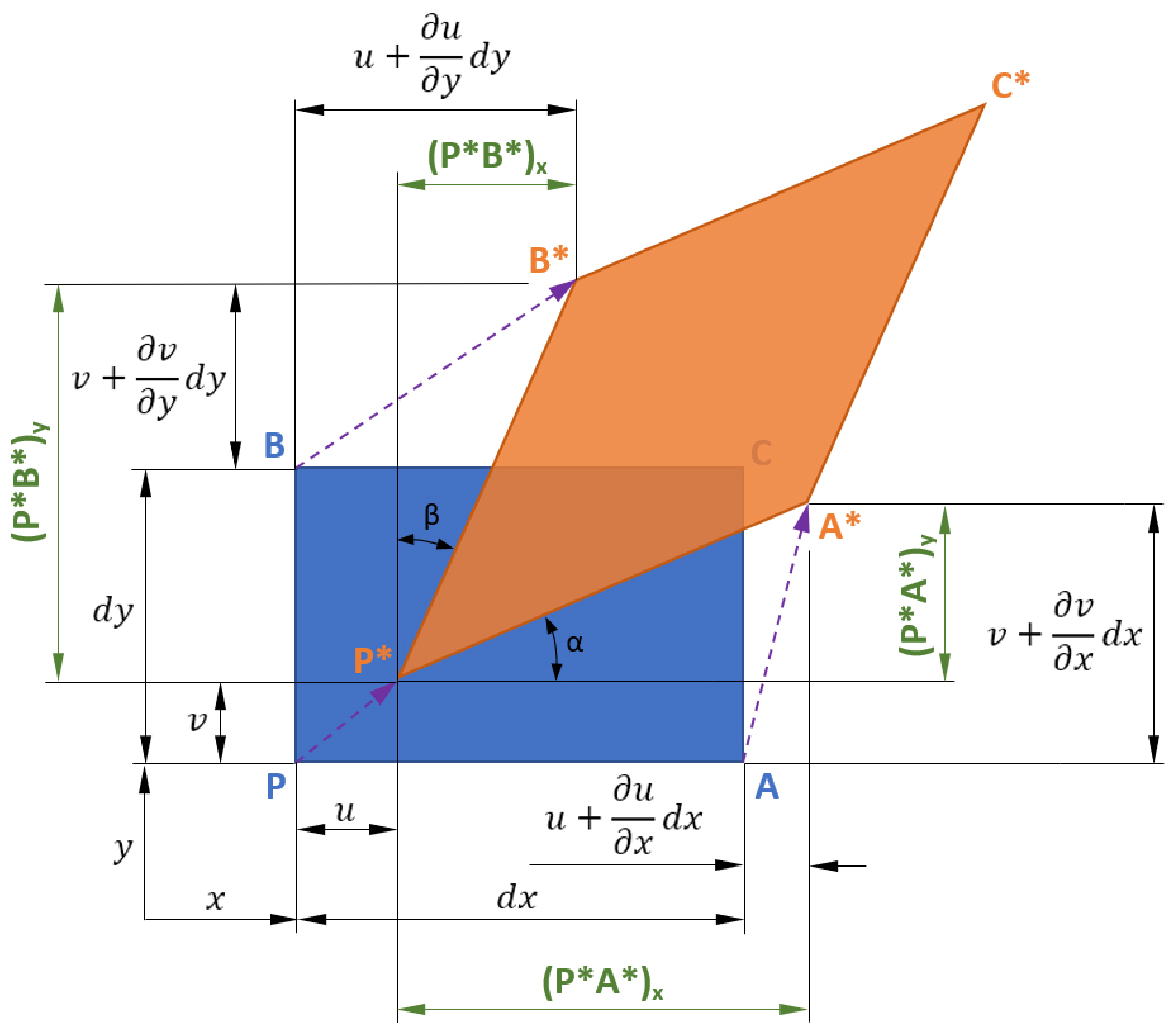
İTÜ MAKİNA FAKÜLTESİ

ADİL YÜCEL

DÜZLEMDE ŞEKİL DEĞİŞTİRME DURUMU



$$(P^*A^*)_x = dx + \left[u + \frac{\partial u}{\partial x} dx\right] - u = dx + \frac{\partial u}{\partial x} dx \qquad (P^*A^*)_y = \left[v + \frac{\partial v}{\partial x} dx\right] - v = \frac{\partial v}{\partial x} dx$$

$$(P^*B^*)_x = \left[u + \frac{\partial u}{\partial y} dy\right] - u = \frac{\partial u}{\partial y} dy \qquad (P^*B^*)_y = dy + \left[v + \frac{\partial v}{\partial y} dy\right] - v = dy + \frac{\partial v}{\partial y} dy$$

$$P^*A^* = (dx)^* = \sqrt{[(P^*A^*)_x]^2 + [(P^*A^*)_y]^2} = \sqrt{\left[dx + \frac{\partial u}{\partial x} dx\right]^2 + \left[\frac{\partial v}{\partial x} dx\right]^2}$$

$$P^*B^* = (dy)^* = \sqrt{[(P^*B^*)_x]^2 + [(P^*B^*)_y]^2} = \sqrt{\left[\frac{\partial u}{\partial y} dy\right]^2 + \left[dy + \frac{\partial v}{\partial y} dy\right]^2}$$

$$(dx)^* = dx\sqrt{1 + 2\left[\frac{\partial u}{\partial x}\right] + \left[\frac{\partial u}{\partial x}\right]^2 + \left[\frac{\partial v}{\partial x}\right]^2} \approx dx\sqrt{1 + 2\left[\frac{\partial u}{\partial x}\right] + \left[\frac{\partial u}{\partial x}\right]^2} = dx\sqrt{\left[1 + \frac{\partial u}{\partial x}\right]^2} = dx\left[1 + \frac{\partial u}{\partial x}\right]$$

$$(dy)^* = dy\sqrt{1 + 2\left[\frac{\partial v}{\partial y}\right] + \left[\frac{\partial v}{\partial y}\right]^2 + \left[\frac{\partial u}{\partial y}\right]^2} \approx dy\sqrt{1 + 2\left[\frac{\partial v}{\partial y}\right] + \left[\frac{\partial v}{\partial y}\right]^2} = dy\sqrt{\left[1 + \frac{\partial v}{\partial y}\right]^2} = dy\left[1 + \frac{\partial v}{\partial y}\right]$$

$$\varepsilon_x = \frac{(dx)^* - dx}{dx} = \frac{(dx)^*}{dx} - 1 = \frac{dx\left[1 + \frac{\partial u}{\partial x}\right]}{dx} - 1 \;\Rightarrow\; \varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_y = \frac{(dy)^* - dy}{dy} = \frac{(dy)^*}{dy} - 1 = \frac{dy\left[1 + \frac{\partial v}{\partial y}\right]}{dy} - 1 \;\Rightarrow\; \varepsilon_y = \frac{\partial v}{\partial y}$$

$$\gamma_{xy} = \alpha + \beta \approx \tan(\alpha) + \tan(\beta) = \frac{(P^*A^*)_y}{(P^*A^*)_x} + \frac{(P^*B^*)_x}{(P^*B^*)_y}$$

$$\gamma_{xy} = \frac{\left[\frac{\partial v}{\partial x} dx\right]}{dx + \left[\frac{\partial u}{\partial x} dx\right]} + \frac{\left[\frac{\partial u}{\partial y} dy\right]}{dy + \left[\frac{\partial v}{\partial y} dy\right]} = \frac{\left[\frac{\partial v}{\partial x}\right]}{1 + \left[\frac{\partial u}{\partial x}\right]} + \frac{\left[\frac{\partial u}{\partial y}\right]}{1 + \left[\frac{\partial v}{\partial y}\right]} \approx \frac{\left[\frac{\partial v}{\partial x}\right]}{1} + \frac{\left[\frac{\partial u}{\partial y}\right]}{1} \;\Rightarrow\; \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$